UNIT – 5

Project Scheduling

Project :

A project is defined as a combination of interrelated activities all of which must be executed in a certain order to achieve a set goal.

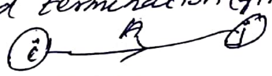
A large and complex project involves usually a number of interrelated activities requiring men, machines and materials.

The three main managerial functions for any project are 1. planning 2. scheduling 3. control.

Basic terminologies :

Activity is a task or an item of work to be done in a project. An activity consumes resources like time, money, labour etc.

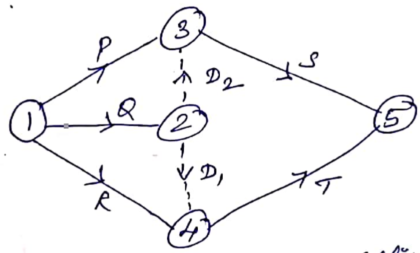
An activity is represented by an arrow with a node (event) at the beginning and a node (event) at the end indicating the start and termination (finish) of the activity.



(arrow diagram)

1. If there are five activities P, Q, R, S and T such that P, Q, R have no immediate predecessors but S and T have immediate predecessors P, Q and Q, R respectively. Represent this situation by a network.

Solution :



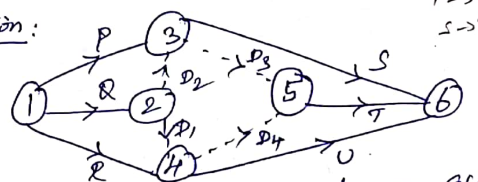
D_1 and D_2 are dummy activities.

(2) Draw the network for the project whose activities and their precedence relationships are given below

Activity :	P	Q	R	S	T	U
Predecessor :	–	–	–	P, Q	P, R	Q, R

P→R→R
S→T→U

Solution :

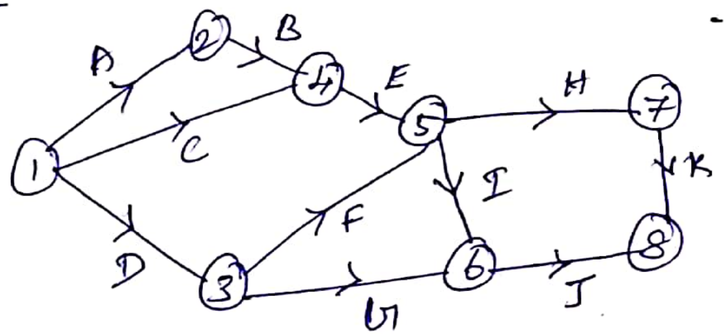


D_1, D_2, D_3, D_4 are dummy activities.

③ Draw the network for the project whose activities with their predecessor relationships are given below:

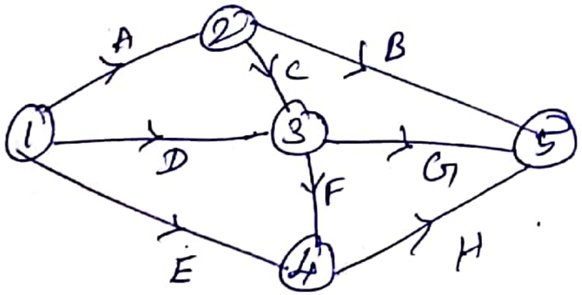
A, C, D can start simultaneously ;
E > B , C ; F, G > D ; H, I > E, F ; J > I, G ;
K > H ; B > A

Soln.:



④ Construct the network for the project whose activities and their relationships are as given below : Activities : A, D, E can start simultaneously
Activities : B, C > A ; G, F > D, C ; H > E, F.

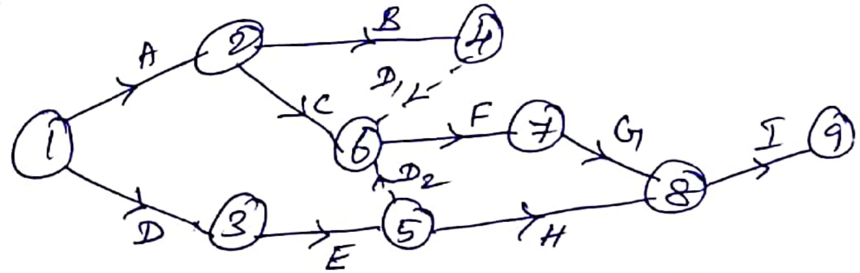
Solution:



⑤ Draw the network for the project whose activities and their precedence relationships are as given below:

Activities :(to)	A	B	C	D	E	F	G	H	I
Immediate Predecessor : (from)		A	A	–	D	B,C,E	F	E	G,H

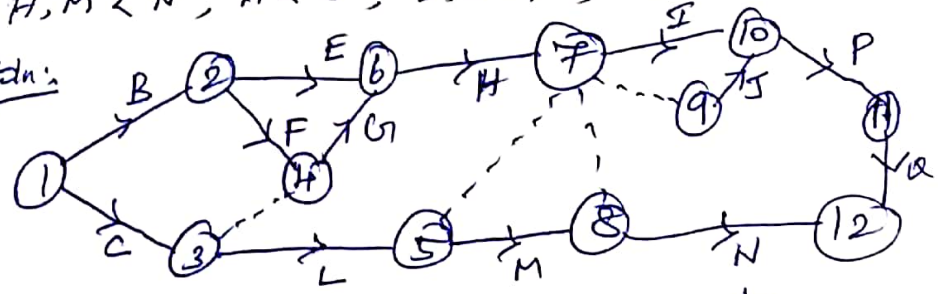
Solution: from A to B. from E to H



D_1 and D_2 are dummy activities.

⑥ Construct the network for the project whose precedence relationships are as given below.
B < E, F ; C < G, L ; E, G < H ; L, H < I ; L < M ;
H, M < N ; A < J ; I, J < P ; P < Q.

Soln.:



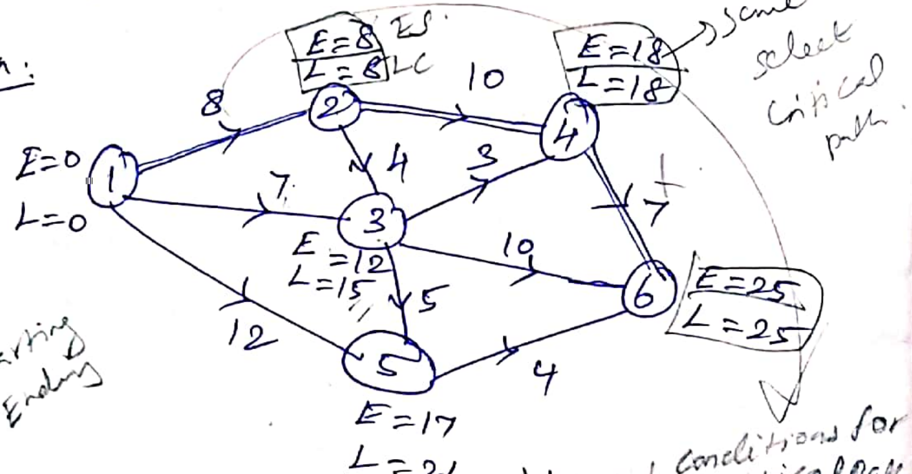
D_1, D_2, D_3 and D_4 are dummy activities.

Critical Path Method

1. Calculate the earliest start, earliest finish, latest start and latest finish of each activity of the project given below and determine the critical path of the project.

Activity	1-2	1-3	1-5	2-3	2-4	3-4	3-5	3-6	4-6	5-6
Duration in (weeks)	8	7	12	4	10	3	5	10	7	4

Soln:



i → Starting
j → Ending

Earliest Starting time (ES_j)

$ES_j = \max_i (ES_i + D_{ij})$

Latest completion time (LC_i)

$LC_i = \min_j (LC_j - D_{ij})$

Conditions for critical path

$ES_i = LC_i$

$ES_j = LC_j$

$ES_j - ES_i =$

$LC_j - LC_i = D_{ij}$

Activity	Duration (in weeks)	Earliest Start (E)	Earliest Finish	Latest Start $LC_j - D_{ij}$	Latest Finish
1-2	8	0	8	0	8
1-3	7	0	7	15-7=8	15
1-5	12	0	12	21-12 = 9	21
2-3	4	8	12	15-4 = 11	15
2-4	10	8	18	8	18
3-4	3	12	15	18-3 = 15	18
3-5	5	12	17	21-5 = 16	21
3-6	10	12	22	25-10 = 15	25
4-6	7	18	25	(25-7) = 18	25
5-6	4	17	21	25-4 = 21	25

Floats:

Total float of an activity (T.F) is defined as the difference between the latest finish and the earliest finish of the activity or the difference between the latest start and the earliest start of the activity.

Total float (T.F) = L.F − E.F

(or) = L.S − E.S.

$T.F = LC_j - ES_i - D_{ij}$

i.e, the time required to complete an activity will have no bearing on the completion times of any other activity of the project.

(ii) The activity durations follow β-distributions. β distribution is a probability distribution with density function $k(t-a)^{\alpha}(b-t)^{\beta}$ with Mean $t_e = \frac{1}{3}\left[2t_m + \frac{1}{2}(t_o - t_p)\right]$ and the Standard deviation $\sigma_t = \frac{t_p - t_o}{6}$.

PERT procedure:

1. Draw the project network
2. Compute the expected duration of each activity $t_e = \frac{t_o + 4t_m + t_p}{6}$
3. Compute the expected variance $\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$ of each activity.
4. Compute the earliest start, earliest finish, latest start, latest finish and total float of each activity.
5. Determine the critical path and identify critical activities.
6. Compute the expected variance of the project length (also called the variance of the critical path) σ_c^2 which is the sum of the variances of all the critical activities.
7. Compute the expected standard deviation of the project length σ_c and calculate the standard normal deviate $\frac{T_S - T_E}{\sigma_c}$ where

T_S = Specified or scheduled time to complete the project.

T_E = Normal expected project duration

σ_c = Expected standard deviation of the project length.

8. Using (7) one can estimate the probability of completing the project within a specified time, using the normal curve (Area) tables.

Note: (2), (3) are valid because of assumption (i)

(ii). (6) is valid because of assumption

PROGRAMME EVALUATION REVIEW TECHNIQUE (PERT)

PERT calculations depend upon the following three time estimates:

Optimistic (least) time estimate: (t_o or a) is the duration of any activity when everything goes on very well during the project. i.e, Labourers are available and come in time, machines are working properly, money is available whenever needed, there is no scarcity of raw material needed etc.

Pessimistic (greatest) time estimate: (t_p or b) is the duration of any activity when almost every thing goes against ~~our~~ on will and a lot of difficulties is faced while doing a project.

Most likely time estimate: (t_m or m) is the

Free Float: Free float of an activity (F.F) is that portion of the total float which can be used for rescheduling that activity without affecting the succeeding activity.

Free float (F.F) = T.F − (L.F − E.F)

Free Float ≤ Total float for any activity.

Independent float (I.F)

I.F of an activity is the amount of time by which the activity can be rescheduled without affecting the preceding or succeeding activities of that activity.

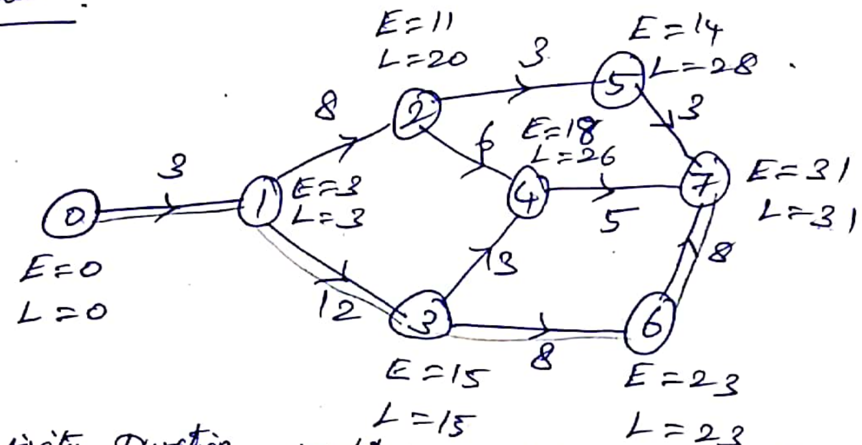
I.F = F.F − (L.F − E.F)

Note: I.F ≤ F.F ≤ T.F

(1) Construct the network for the project whose activities are given below and compute the total, free and independent float of each activity and hence determine the critical path and the project duration.

Activity	0-1	1-2	1-3	2-4	2-5	3-4	3-6	4-7	5-7	6-7
Duration (in weeks)	3	8	12	6	3	3	8	5	3	8

Solution:



Activity	Duration (in weeks)	Earliest Start E.S	Earliest Finish E.F	Latest Start L.S	Latest Finish L.F	Floats TF	FF	IF
0-1	3	0	3	0	3	0	0	0
1-2	8	3	11	12	20	9	0	0
1-3	12	3	15	3	15	0	0	0
2-4	6	11	18	20	26	9	1	−8
2-5	3	11	14	25	28	14	0	−9
3-4	3	15	18	23	26	8	0	0
3-6	8	15	23	15	23	0	0	0
4-7	5	18	23	26	31	8	8	0
5-7	3	14	18	28	31	14	14	0
6-7	8	23	31	23	31	0	0	0

Critical path is 0-1-3-6-7. Project duration = 31 weeks.

(i) two months later than expected

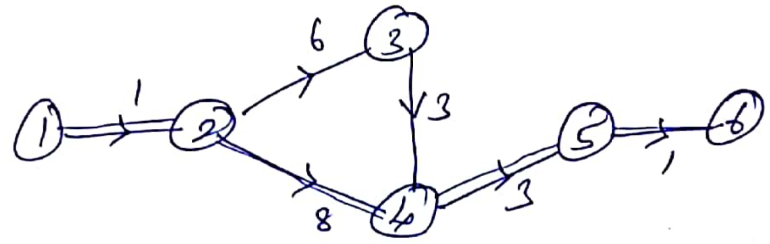
(ii) not more than 3 months earlier than expected

(iii) what due date has about 90%. Chance of being met?

Soln: a)

Activity	a	m	b	te	σ
1-2	0.8	1.0	1.2	1	0.067
2-3	3.7	5.6	9.9	6	1.03
2-4	6.2	6.6	15.4	8	1.53
3-4	2.1	2.7	5.1	3	0.5
4-5	0.8	3.4	3.6	3	0.47
5-6	0.9	1.0	1.1	1	0.033

b)



c) critical path : 1-2-3-4-5-6

Expected project length = 14 months

Expected variance $= (0.067)^2 + (1.03)^2 + (0.5)^2 + (0.47)^2 + (0.033)^2$

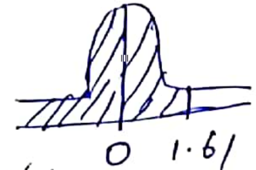
$= 1.5374$

$\sigma_c = \sqrt{1.5374} = 1.2399$

d) (i) $T_S = 16$, $T_E = 14$, $\sigma_c = 1.2399$

$Z = \frac{T_S - T_E}{\sigma_c} = \frac{16-14}{\sigma_c} = \frac{2}{1.2399} = 1.61$

$P(T_S \le 16) = 0.5 + 0.4463 = 0.9463$



$P(T_S \le 16) = 94.63\%$

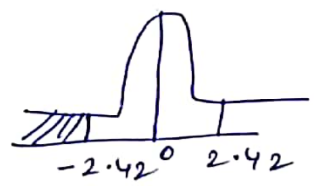
= Required probability

(ii) $T_S = 11$, $T_E = 14$, $\sigma_c = 1.2399$

$Z = \frac{T_S - T_E}{\sigma_c} = \frac{11-14}{1.2399} = \frac{-3}{1.2399} = -2.42$

$P(T_S \le 11) = 0.5 - 0.4922$

$= 0.0078$



∴ Required probability = 0.78 %.

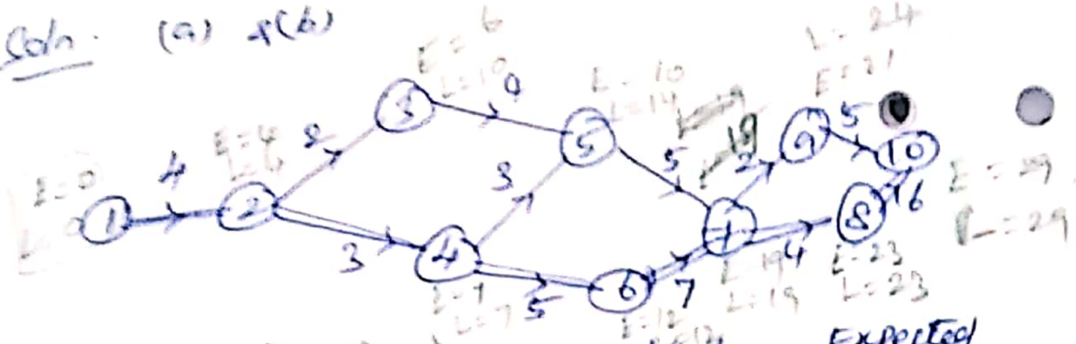
(iii) Z value for 90%. area in the table = 1.28 nearly.

Let T_S be the required due date. Then $Z = \frac{T_S - T_E}{\sigma_c}$

i.e., $1.28 = \frac{T_S - 14}{1.2399}$

$T_S = 14 + 1.28(1.2399) = 15.59$ months nearly.

Soln. (a) & (b)



Activity	t_o (a)	t_m (m)	t_p (b)	Expected duration $t_e = \frac{t_o + 4t_m + t_p}{6}$	Expected variance $\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$
1-2	3	4	5	4	$1/9 = 0.11$
2-3	1	2	3	2	$1/9 = 0.11$
2-4	2	3	4	3	$1/9 = 0.11$
3-5	3	4	5	4	$1/9 = 0.11$
4-5	1	3	5	3	$4/9 = 0.44$
4-6	3	5	7	5	$4/9 = 0.44$
5-7	4	5	6	5	$1/9 = 0.11$
6-7	6	7	8	7	$1/9 = 0.11$
7-8	2	4	6	4	$4/9 = 0.44$
7-9	1	2	3	2	$1/9 = 0.11$
8-10	4	6	8	6	$4/9 = 0.44$
9-10	3	5	7	5	$4/9 = 0.44$

Critical Path 1-2-4-6-7-8-10

Expected Project duration = 29 weeks

c) Expected variance of the project length = Sum of the expected variances of all the critical activities

$$= 1/9 + 1/9 + 4/9 + 1/9 + 4/9 + 4/9 = \frac{15}{9} = 1.67$$

2. Three time estimates (in months) of all activities of a project are as given below

Time in months

Activity	a	m	b
1-2	0.8	1.0	1.2
2-3	3.7	5.6	9.9
2-4	6.2	6.6	15.4
3-4	2.1	2.7	6.1
4-5	0.8	3.4	3.6
5-6	0.9	1.0	1.1

a) Find the expected duration and standard deviation of each activity.

b) Construct the project network

c) Determine the critical path, expected project length and expected variance of the project length.

d) What is the probability that the project will be completed

1. Construct the network for the project whose activities and the three time estimates of these activities (in weeks) are given below. Compute a) Expected duration of each activity.

b) Expected variance of each activity

c) Expected variance of the project length.

Activity	t_o	t_m	t_p
1-2	3	4	5
2-3	1	2	3
2-4	2	3	4
3-5	3	4	5
4-5	1	3	5
4-6	3	5	7
5-7	4	5	6
6-7	6	7	8
7-8	2	4	6
7-9	1	2	3
8-10	04	6	8
9-10	3	5	7